($\vec{v}$ here is the "orthogonal projection of $\vec{b}$") soln

Math 2D Quiz 2 Morning - January 12, 2016

Please put name and ID on *both* sides for grading and redistribution!

Show all of your work. *There is a question on the back side.

1pt each

1. (a) Give the definition of a vector projection of $\mathbf{b}$ onto $\mathbf{a}$. In other words, express $\text{proj}_{\mathbf{a}}\mathbf{b}$.

(b) Let $\mathbf{a} = <1,4,0>$, $\mathbf{b} = <2,3,0>$. Compute the vector $\mathbf{v} = \mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$.

(c) Compute $\mathbf{b} \times \mathbf{a}$.

(d) What is $\mathbf{v} \times \mathbf{a}$? You must either directly compute this, or explain your answer.

(d) Determine if $\mathbf{v}$ is perpendicular to $\mathbf{a}$. Justify your answer.

Based on 12.3.45,46

a) $\text{proj}_{\vec{a}}\vec{b} = \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a}$

b) First, $\vec{a}\cdot\vec{b} = 1(2) + 4(3) = 14$; $|\vec{a}|^2 = \sqrt{1+16}^{\,2} = 17$

so, $\text{proj}_{\vec{a}}\vec{b} = \dfrac{14}{17}<1,4,0> = \boxed{<\dfrac{14}{17}, \dfrac{56}{17}, 0>}$

Thus, $\vec{v} = <2,3,0> - <\dfrac{14}{17}, \dfrac{56}{17}, 0> = \boxed{<\dfrac{20}{17}, -\dfrac{5}{17}, 0>}$

c) $\vec{b}\times\vec{a} = \det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 1 & 4 & 0 \end{pmatrix} = \det\begin{pmatrix} 3 & 0 \\ 4 & 0 \end{pmatrix}\hat{i} - \det\begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}\hat{j} + \det\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\hat{k}$

$= (8-3)\hat{k} = \boxed{5\hat{k}}$

Or, Using $\vec{b}\times\vec{a} = (b_2a_3 - b_3a_2)\hat{i} + (b_3a_1 - b_1a_3)\hat{j} + (b_1a_2 - b_2a_1)\hat{k}$

☆ Note $b_3 = 0$, $a_3 = 0$ so $\vec{b}\times\vec{a} = (8-3)\hat{k} = 5\hat{k}$ ✓

d) $\boxed{\vec{v}\times\vec{a} = \vec{b}\times\vec{a} = 5\hat{k}}$ too, since $\vec{v} = \vec{b} - (\text{scalar})\vec{a}$ and $\vec{a}\times\vec{a} = 0$. (always)

So, $\vec{v}\times\vec{a} = \vec{b}\times\vec{a} - \left(\dfrac{14}{17}\right)\underbrace{\vec{a}\times\vec{a}}_{\text{zero!}} = \vec{b}\times\vec{a} = 5\hat{k}$

e) $\vec{v}\cdot\vec{a} = <\dfrac{20}{17}, -\dfrac{5}{17}, 0> \cdot <1,4,0> = \dfrac{20}{17} + \dfrac{4(-5)}{17} = 0$

Thus, since $\vec{v}\cdot\vec{a} = 0 \iff \boxed{\vec{v}\perp\vec{a}}$ (yes it is perpendicular)

2. (a) Graph the region in $\mathbb{R}^3$ represented by

2pts

$$x^2 + y^2 \leq 1, \quad 0 \leq z \leq 2.$$

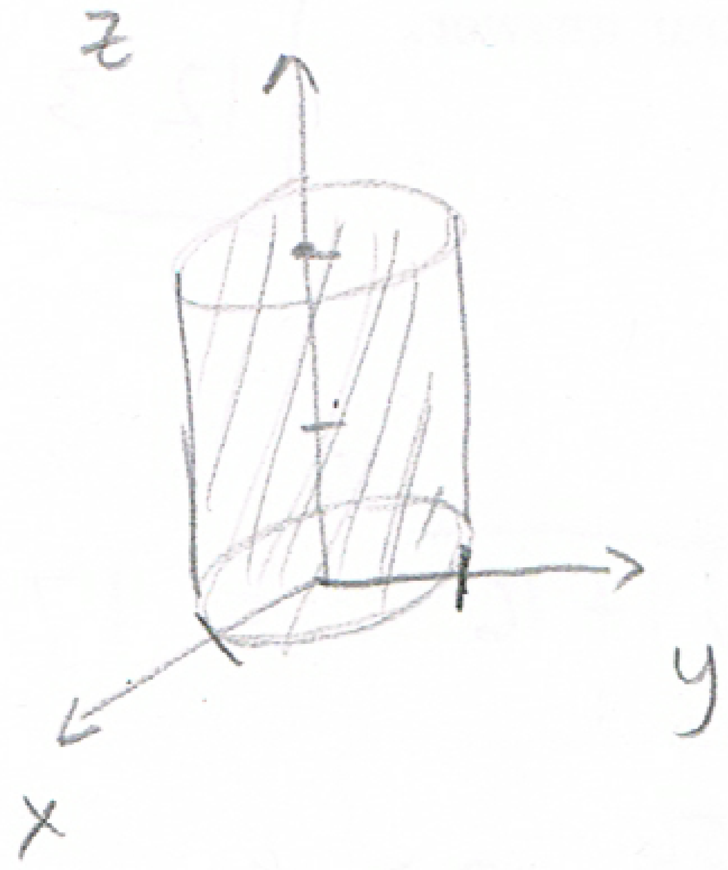
Based on 12.1.37

Describe the region that you graphed - be brief.

3pts. (b) If $\mathbf{a} \cdot \mathbf{b} = -7$ and $\mathbf{a} \times \mathbf{b} =< 2, 3, 6 >$, determine the angle between vectors $\mathbf{a}$ and $\mathbf{b}$.

Based on 12.4.43

a)



Solid cylinder of radius 1

and height 2.

(Between $z=0$ and $z=2$ planes)

↑ xy plane

b) We know $\vec{a} \cdot \vec{b} = |a||b| \cos\theta_{ab}$

$|\vec{a} \times \vec{b}| = |a||b| \sin\theta_{ab}$ // Divide

Thus, $\tan\theta_{ab} = \dfrac{|\vec{a} \times \vec{b}|}{\vec{a} \cdot \vec{b}}$. Here, $|\vec{a} \times \vec{b}| = \sqrt{4+9+36} = \sqrt{49} = 7$

$\vec{a} \cdot \vec{b} = -7$,

so $\tan\theta_{ab} = \dfrac{7}{-7} = -1 \implies \boxed{\theta_{ab} = \dfrac{3\pi}{4} \text{ or } -\dfrac{\pi}{4}}$

$(135^\circ$ or $-45^\circ)$

☆ Since $\vec{a} \cdot \vec{b}$ is negative, θ_{ab} must be greater than 90° $(\frac{\pi}{2})$ in magnitude, so $\boxed{\theta_{ab} = \frac{3\pi}{4}}$ here.

↳ Negative Dot Product Implies the vectors are facing "away" from each other.

Ex.